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Identity, Leibniz's Law and Non-transitive Reasoning

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Abstract Arguments based on Leibniz's Law seem to show that there is no room for either indefinite or contingent identity. The arguments seem to prove too much, but their conclusion is hard to resist if we want to keep Leibniz's Law. We present a novel approach to this issue, based on an appropriate modification of the notion of logical consequence.

Keywords Identity · Contingent identity · Indefinite identity · Leibniz's Law · Logical consequence · Nontransitive reasoning

1 Introduction

An identity statement is a sentence of the form '*a* is *b*' where both *a* and *b* are singular terms. In his *Conceptual Notation*, Frege argues that the inclusion of a symbol for identity is not a pointless matter in the language for pure thought. The reason, he says, is that identity statements convey the information that the denotation of a singular

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term *a* is the same as the denotation of a singular term *b* where *a* and *b* are associated to different ways of presentation of an object and, thus, different contents. A true identity statement of this sort is synthetic in the Kantian sense (Frege 1879). In 'Sense and reference' Frege develops this idea further by arguing that the cognitive value of identity statements is explained by the *senses* associated to expressions, objective entities that are different and (to a certain extent) independent of the reference of an expression. A singular term denotes, if it does, an object and expresses a sense. In order to make a case for this view on senses, Frege argues that in indirect discourse there is a shift of reference so that subordinate clauses take their senses as reference.

> "That in the cases of the first kind the referent of the subordinate clause is in fact the thought can also be recognized by seeing that it is indifferent to the truth of the whole whether the subordinate clause is true or false. Let us compare, for instance, the two sentences "Copernicus believed that the planetary orbits are circles" and "Copernicus believed that the apparent motion of the sun is produced by the real motion of the earth." One subordinate clause can be substituted for the other without harm to the truth. The main clause and the subordinate clause together have as their sense only a single thought, and the truth of the whole includes neither the truth nor the untruth of the subordinate clause. In such cases it is not permissible to replace one expression in the subordinate clause by another having the same customary referent, but only by one having the same indirect referent, i.e., the same customary sense" (Frege 1892).

(In what follows, we adopt the Fregean sense talk for simplicity, though nothing commits us to that particular view of indirect reference).

There are two interesting points in these considerations: first, the failure of substitutivity is the hallmark that senses, and not ordinary references, are at issue; secondly, according to Frege expressions in subordinate clauses always take their senses as reference. Now this second point must be qualified. There are various cases in which an expression in a subordinate clause can still denote its ordinary reference. For example,

(a) Ralph believes that the lover of his wife is his most loyal friend.

The description 'the lover of his wife' is meant to be read purely *de re* or *referentially* in this context. The singular term denotes its ordinary reference even if it appears in the subordinate clause (under the scope of "believes"). A similar remark can be applied to identity statements. It might be the case that we ordinarily understand identity statements as reporting something involving the sense of expressions, but we might likewise understand them in a purely *referential way* (following Quine's terminology, see Quine 1960, Recanati 2000).

Implicit in Frege's discussion is the idea that this second way in which we might understand identity statements would make completely pointless the introduction of an identity symbol and of different terms for the same object. Identity statements in this second reading would be analytic in the Kantian sense. Although identity statements in the second sense are somehow more fundamental (as they seem to be involved in the content of identity statements in the previous sense), they are analytic and so, fully determinate and necessary. Identity in this sense might be more fundamental, but it raises no big issue. Or so it seems.

Suppose we have a language L with one-place predicates $A(x)$ and an identity symbol '=' and consider the following principle and inference rule (where '→' is a conditional, '∧' a conjunction, '⊢' a symbol for consequence and t, u singular terms of L):

$$\vdash (A(t) \wedge t = u) \rightarrow A(u)$$
$$A(t), t = u \vdash A(u)$$

Should we accept such schemata for any appropriate substitution of predicates and terms of L? Well, this depends on what kind of language L is and what reading we are giving to identity statements in L. Under the first reading of identity, the principle and inference rule express the *substitutivity* of co-referential terms. We are willing to accept these for *extensional* languages, but not otherwise. For suppose we have the expression 'necessarily' in our language (as a sentential operator '□'):

$$\Box(8 > 7) \wedge 8 = \text{The number of planets} \nvdash \Box(\text{The number of planets} > 7)$$

This inference is felt as clearly invalid when we read identity in the non-referential way. Under the referential reading, on the other hand, the truth conditions of identity statements should not depend on what sort of language we are dealing with. Under the referential understanding, the principle and inference above express Leibniz's Law according to which identical objects share all properties (at least, all properties that can be expressed in L). There is a mutual dependence here: identity statements in the referential reading justify the acceptance of the principle and inference above, and at the same time, the acceptance of the principle and inference above *in any context* is a hallmark of the referential reading of identity statements.

This paper is concerned with identity statements under the purely *de re* or referential reading of singular terms. Our question is whether such statements can be indefinite or contingent. We wil investigate not just whether there are indefinite and contingent identity statements (uncontroversial) but whether, under the referential reading, an identity statement might be indefinite or contingent (controversial).

In Section 2, we first present some prima facie cases for indefinite and contingent identity. Then, we rehash a couple of arguments against indefinite and contingent involving Leibniz's Law. In Section 3, we present our reasons for and against each contender in this debate and try out an analysis inspired by our previous work on vagueness and truth.

2 Arguments from Leibniz's Law

2.1 Some Prima Facie Cases for Indefinite and Contingent Identity

Indefinite Identity Priest's (2010) motorbike (Theseus' ship):

> […] suppose I change the exhaust pipes on my bike; is it or is it not the same bike as before? It is, as the traffic registration department and the insurance company will testify; but it is not, since it is manifestly different in appearance, sound, and acceleration […] A standard reply here is to distinguish between the

> bike itself and its properties. After the change of exhaust pipes the bike is numerically the same bike; it is just that some of its properties are different. Perhaps, for the case at hand, this is the right thing to say. But the categorical distinction between the thing itself and its properties is one which is difficult to sustain; to suppose that the bike is something over and above all of its properties is simply to make it a mysterious *Ding an sich*. Thus, suppose that I change, not just the exhaust pipes, but, in succeeding weeks, the handlebars, wheels, engine, and in fact all the parts, until nothing of the original is left. It is now a numerically different bike, as even the traffic office and the insurance company will concur. At some stage, it has changed into a different bike, i.e. it has become a different machine: the bike itself is numerically different. (This is a variation on the old problem of the ship of Theseus.) (Priest 2010, p. 406)

The literature is crowded with other examples with more or less intuitive appeal like the fission and fusion of amoebas and brain transplants (see Williams 2007, pp. 140–141; Williams 2008, pp. 763–765; Magidor 2011).

Contingent Identity Gibbard's (1975) Goliath

Consider Gibbard's (1975) famous example of a statue, named 'Goliath', and the clay out of which it is composed, named 'Lumpl'. It is natural to assume that the statement 'Lumpl = Goliath' is true. But Lumpl, the piece of clay, might have been rolled into a ball and turned into a new, very different statue, named 'David'. But it is impossible for Goliath to be identical to David. Thus, in the possible world in which 'Lumpl = David', 'Lumpl ≠ Goliath'. Therefore, Lumpl is identical to Goliath but only contingently so.

2.2 Evans' Argument Against Indefinite Identity Statements

In his 1978 paper, Gareth Evans presents an argument against the claim that there might be vague objects or, more precisely, against the claim that there might be indefinite identity statements.

Suppose '$\Box A$' means 'It is definite that'. '∇A' is an abbreviation for '$\neg\Box A \wedge \neg\Box\neg A$' (it is indefinite whether A), and thus, 'ΔA' means the same as '$\neg\nabla A$' (it is definite whether A). Now, assume that there is an indefinite identity statement. Evans' argument runs as follows:

(1)	$\nabla(a = b)$	(assumption)
(2)	$\lambda x[\nabla(x = a)]b$	(abstraction from 1)
(3)	$\neg\nabla(a = a)$	(truism, since a = a is a logical truth!)
(4)	$\neg\lambda x[\nabla(x = a)]a$	(abstraction from 3)
(5)	$\neg(a = b)$	(from 2 and 4 by contraposition and Leibniz's Law)

As Lewis (1988) points out, there is a sense in which the argument is fallacious, since one might argue that abstraction in the scope of '∇' is not valid (in much the same way in which abstraction in a 'contingency' operator is not valid). The interesting point is that this way of blocking the argument is available to the defender of semantic indeterminacy, but not to the defender of *ontic* indeterminacy (pace Williams 2008). For the defender of ontic indeterminacy, the terms a and b in the argument determinately denote an intrinsically indeterminate object. We should add that, as we point out below, similar reasons prevent blocking the argument appealing to a failure of Leibniz's Law (Parsons and Woodruff 1995, for example, give up on the contrapositive of Leibniz’s Law). Since Leibniz's Law is the hallmark of the understanding of an identity statement under its referential reading, its failure raises suspicion on whether our understanding of the indeterminacy is really ontic indeterminacy.

It is interesting to point out, for what comes later, that we could add a last step to Evans' argument applying *conditional proof* (although this rule must be applied with some care in the presence of modalities, its application in this case is unproblematic):

(6) $\vdash \nabla(a=b) \rightarrow \neg(a=b)$
whose contrapositive form is

(7) $\vdash (a=b) \rightarrow \Delta(a=b)$
In words, if $a = b$, then it is determinate whether $a = b$.

2.3 Argument Against Contingent Identity Statements

There is a famous argument due to Barcan Marcus and Kripke, and defended by Wiggins (1980) and Williamson (1996) to the effect that the identity of an object is a necessary matter:

(8)	$(x{=}y)\vdash(x{=}x)$	(logic!)
(9)	$(x{=}y)\vdash\Box(x{=}x)$	(necessitation)
(10)	$(x{=}y)\vdash\lambda z\Box(x{=}z)\ (x)$	(abstraction)
(11)	$(x{=}y)\vdash\lambda z\Box(x{=}z)\ (y)$	(Leibniz's Law)
(12)	$(x{=}y)\vdash\Box(x{=}y)$	(reduction)
(13)	$\vdash(x{=}y)\rightarrow\Box(x{=}y)$	(conditional proof)[1]

The argument proceeds in much the same way as Evans'. Though we wrote Evans' argument in non-sequent style (to be faithful to the original argument), as pointed out above, Evans' argument allows us to conclude (7)

Suppose now that both modalities obey a logic as strong as normal modal logic **T** (this is a reasonable assumption for each reading of the modality). Then (7) and (16) are formally equivalent:

(7)	$\vdash (a=b) \rightarrow \Delta(a=b)$	
(14)	$\vdash (a=b) \rightarrow (\Box(a=b) \vee \Box\neg(a=b))$	(unpacking definition of 'Δ')
(15)	$\vdash (a=b) \rightarrow \neg\Box\neg(a=b)$	(instance of T : $\Box A \rightarrow A$)
(16)	$\vdash (a=b) \rightarrow \Box(a=b)$	(from 14, 15 by disjunctive syllogism)

Conversely, (16) entails (14) [and so, (7)] since $\Box$ $(a = b)$ is classically stronger than $\Box(a=b)\vee\Box\neg(a=b)$.

A principle like (15) joint with a logic as strong as **T** trivializes the modality over identity statements (see Hughes and Cresswell 1996, p. 64). That is, we can delete, preserving logical equivalence, the modalities in formulas containing only identity statements (or, at any rate, for formulas with modalities with just identity statements

[1] In general, it might be risky to apply conditional proof after the rule of necessitation. In the present case, however, there is no problem since necessitation is applied to something that can be derived independently of the premises discharged in the application of conditional proof.

under their scope). Suppose we wanted to say that some objects are indeterminately identical or that they are contingently identical:

(17) $\nabla(a = b)$
(18) $(a = b) \wedge \neg\Box(a = b)$

(17) unpacks into $\neg\Box(a = b) \wedge \neg\Box\neg(a = b)$ and, deleting the modalities, we get $\neg(a = b) \wedge \neg\neg(a = b)$. Similarly, (18) would be equivalent to $(a = b) \wedge \neg(a = b)$. So unless we adopt a dialetheist point of view, allowing for true contradictions these arguments lead to fatal conclusions for the defender of indeterminate or contingent identity.

3 Non-transitive Reasoning

In this section, we present our view on indeterminate and contingent identity. This view is inspired by our account of the paradoxes of vagueness and the Liar (in our papers: Cobreros et al. 2012a, 2013a, b and c). Before getting into the details, however, let us take stock of the discussion so far.

In Section 2, we presented some prima facie plausible cases for indeterminate and contingent identity. Then, we saw that, based on Leibniz's Law and some minimal amount of logic, we can 'prove' that identity statements are all necessary and determinate. So we are faced with the following dilemma: either we agree that, contrary to initial appearances, there is no indefinite or contingent identity, or we resist this conclusion by giving up Leibniz's Law. But we cannot feel satisfied with either horn of the dilemma.

On the one hand, giving up Leibniz's Law is dialectically unsatisfying since it raises the suspicion that we are not reading identity statements in a referential way (recall that the validity of substitution is one of the hallmarks of the referential understanding of identity). So we can abandon Leibniz's Law, but this deprives identity statements of their intended referential reading. On the other hand, we find very surprising that we can 'prove' a claim about the world of that sort on such a narrow basis. As Williamson points out, Wiggins' argument shows that

> [...] identity is a metaphysically rigid relation. Either it necessarily and determinately relates a given pair of individuals, or it necessarily and determinately fails to relate them. In that sense, the facts about it form part of the necessary and determinate structure of reality (Williamson 1996, p. 2).

We find ourselves in a paradox-like scenario: we seem to be committed to unwanted conclusions from apparently uncontroversial premises. In this sort of situation, there is a standard reaction: 'revise your logic'. However, this revision can be done in, broadly speaking, two different ways.

The first approach is to weaken the logic by rejecting some classically valid *inferences*, that is, by rejecting the validity of some statements of the form ($\Gamma \vdash \Delta$) involved in the derivation of unwanted conclusions. Some reactions to the Liar paradox, like those based on K3, involve the rejection of *excluded middle* ($\top \vdash A \vee \neg A$) whereas others, like those based on LP, involve the rejection of *explosion* ($A \wedge \neg A \vdash \bot$). This sort of weakening comes at the price of losing at the same time other inferences that we would like to keep, like *identity* ($\top \vdash A \rightarrow A$) in the case of K3 and modus ponens ($A \rightarrow B$, $A \vdash B$) in the case of LP. An inference is a relation between formulas; a *meta*inference is a relation between inferences. Not just inferences, but also some metainferences are considered as part and parcel of classical logic, such as the deduction theorem:

$$\Gamma, A \vdash B \Rightarrow \Gamma \vdash A \rightarrow B$$

Naturally, the loss of inferences has an impact on *the relations* between inferences. For example, the deduction theorem does not hold in either K3 or LP.

The second approach is to concentrate directly on metainferences.[2] This has the advantage of allowing us to keep all classically valid inferences while avoiding some problems like, for example, the excessive weakness of K3's or LP's material conditional. One way to achieve this effect of losing *only* some classical metainferences is by allowing different conditions of satisfaction for premises and conclusions. For example, in our paper (Cobreros et al. 2013a), we use a three-valued semantics and define a counterexample as a model where all the premises take value 1 and all the conclusions value 0 (compare this definition with either of K3 or LP; see Priest 2008, pp. 122–125). Since there is no unique value in these models that can be identified unambiguously as 'the notion of truth' preserved by this consequence relation, we tend to favour an *inferentialist* (rather than a *referentialist*) reading of truth values (see Ripley 2013; Cobreros et al. 2013d). This is the approach we take below, with a small proviso to account for contingent identity.

3.1 Indefinite Identity

Let L be a language with first-order variables and constants, one-place predicates, conjunction ($\wedge$), negation ($\neg$) and universal first-order quantifier ($\forall$). An MV-model M is a structure $<D, I>$ where D is a non-empty domain and I an interpretation function satisfying:

- $I(t) \in D$ for t, a constant or variable
- $I(P) \in \{1, \frac{1}{2}, 0\}^D$ (a function from the domain to the set of three values)
- $I(Pt) = I(P)\, I(t)$
- $I(\neg A) = 1 - I(A)$
- $I(A \wedge B) = \min(A, B)$
- $I(\forall xA) = \min(\{I'(A) : I' \text{ is an } x - \text{variant of } I\})$

Other connectives like disjunction ($\vee$), the conditional ($\rightarrow$) and the existential quantifier ($\exists$) are defined in the standard way. These three-valued models are known as strong Kleene models. What makes them interesting is our particular definition of logical consequence. Logical consequence can be defined as absence of countermodels. But in a three-valued semantics, there is some room as to what counts as a countermodel. One might consider that countermodels are models where all the premises take value 1 and all the conclusions value less than 1, or models where all the premises take value more than 0 and all the conclusions take value 0. Depending on the case, we get different weakenings of classical logic: K3 and LP, respectively (again, see Priest 2008, pp. 122–125). We can consider, however, that countermodels are those where all the

[2] Schechter (2011) calls a logic 'weakly classical' if it preserves all classically valid inferences (though not, perhaps, all classically valid metainferences). Our approach to truth and vagueness weakly classical in this sense although, strictly speaking, it yields classical logic for the classical vocabulary and an *extension* of classical logic for languages enjoying transparent truth or similarity predicates. See Ripley (2012) for proof-theoretic results on the theory of transparent truth.

premises take value 1 and all the conclusions value 0. This is the logical consequence relation we call 'ST' (if all the premises are Strictly true, then some conclusions are Tolerantly true). That is, ST contains valid arguments that allow for 'a drop' of truth value.

Surprisingly, for a standard vocabulary, ST consequence is just classical logic (see Cobreros et al. 2012a, 2013a). This logic becomes interesting, however, once we allow expressions that are sensitive to the middle value. In that case, ST becomes *stronger* than classical logic, allowing for new inferences but also leading to failures of some *metainferences*. In the case at hand, in particular, we can define identity in the Leibnizian way in a second-order version of ST:

$$(x = y) \stackrel{\text{def}}{=} \forall P(Px \equiv Py) \quad (\text{the } ' \equiv ' \text{ is ST's material biconditional})$$

To handle definiteness, we assume that $I(\Box(a{=}b))$ equals 1 if for every relevant property P, $I(Pa){=}I(Pb)$, and that it is equal to 0 otherwise (see Cobreros et al. 2013b for details). The result is the following: Leibniz's Law is admissible either as a rule or as an axiom (and in full generality: for any substitution of the full vocabulary). In particular, the following is ST-valid:

$$x = y \rightarrow \Box\, x = y$$

However, the modality does not trivialize, nor does Evans' argument show that indefinite identity is impossible. Recall again the steps in Evans' argument:

(1)	$\nabla(a = b)$	(assumption)
(2)	$\lambda x[\nabla(x = a)]b$	(abstraction from 1)
(3)	$\neg\nabla(a = a)$	(truism, since a = a is a logical truth!)
(4)	$\neg\lambda x[\nabla(x = a)]a$	(abstraction from 3)
(5)	$\neg(a = b)$	(from 2 and 4 by contraposition and Leibniz's Law)

As Evans himself suggests, assuming that premises are definite, we would be able to add a further line to the argument,

(6) $\neg\nabla(a = b)$

thereby showing the impossibility of indefinite identity. Each step in the argument is ST-valid. However, from those premises, (5) takes value ½ and (6) value 0. Thus, even though each step is valid when taken separately, we cannot validly conjoin premises in this case. That is, Evans' argument is, according to the logic ST, an example of failure of the metainference of transitivity.

3.2 Contingent Identity

We now look at the case of contingent identity. This time, instead of considering three-valued models, it is reasonable to concentrate on a possible worlds semantics.[3] We will define our semantics first. In the second place, we will recall the intended features of the target theory of identity. In the third place, we give the definition of logical consequence (this is where all the action is) and conclude by evaluating its features.

[3] In our 2012b paper, it is shown how to define analogues of the non-transitive ST logic for possible world semantics. We shall make some simplifying assumptions below, like that all atomic formulas are of the form $x = y$. We leave the study of more general options for future research.

(a) Semantics

Let L be a first-order language with a single relation symbol '=' as the unique element of the non-logical vocabulary so that the set of atomic sentences ATOM is made out of sentences of the form $x=y$, where x and y are individual variables. Suppose L contains, in addition, modal expressions $\Box$ and $\Diamond$. An interpretation M for this language is a triple $< W, R, [\![\ ]\!] >$:

- $W \subseteq \wp(\text{ATOM})$ (that is, W is a set of sets of atomic sentences). We refer to sets in W with a (small) w. For any w in W:
  - $x=x$ is in w (for any individual variable x)
  - if $x=y$ is in w, then $y=x$ is in w
  - If $x=y$ is in w and $x=z$ is in w, then $x=z$ is in w
- R is a relation in W
- $[\![\ ]\!]$ is a function from sets of sentences to sets of sentences of L satisfying the following conditions:
  - $x=y$ is in $[\![w]\!]$ iff $x=y$ is in w
  - $\neg A$ is in $[\![w]\!]$ iff A is not in $[\![w]\!]$
  - $(A \wedge B)$ is in $[\![w]\!]$ iff both A and B are in $[\![w]\!]$
  - $\forall xA$ is in $[\![w]\!]$ iff for every variable y, $A[x/y]$ is in $[\![w]\!]$ (where $A[x/y]$ is the result of substituting all free occurrences of x in A by y)
  - $\Box A$ is in $[\![w]\!]$ iff for all w': if wRw' then A is in $[\![w]\!]$

Other connectives, including $\exists$ and $\Diamond$, can be defined in the standard way. The idea is, of course, that a formula A 'holds' in a world w just in case that formula is in the set $[\![w]\!]$.

(b) Features of the target theory

We seek to define a theory of identity in which Leibniz's Law is valid, that is classical (at any rate, as classical as possible: identity must be symmetric, reflexive and transitive) and in which contingent identity is consistent. In addition, we would like to keep the underlying logic as classical as possible.

There is a tension here that seems to be plainly impossible to overcome in a fully classical logic. Classical logic is self-dual in the sense that an inference is classically valid just in case the *contraposed* inference is classically valid. In symbols:

$$(\text{Self}-\text{duality}) \ \ \Gamma \vDash \Delta \Leftrightarrow \neg\Delta \vDash \neg\Gamma \quad (\text{where } '\neg\Delta' \text{ means : } '\text{attach} \neg \text{ in front of the deltas}')$$

So for example, the inference *excluded middle* ($\top \vdash A \vee \neg A$) is classically equivalent to *explosion* ($A \wedge \neg A \vdash \bot$). Now we said we would like to keep Leibniz's Law, and we want identity to be reflexive. This means that we want the following:

$$\top \vdash x = y \rightarrow \Box x = y$$

but requiring contingent identity to be consistent is rejecting the validity of the contraposed inference:

$$a = b \wedge \diamond\neg(a = b) \nvdash \bot$$

That is, our logic cannot be self-dual. Although this departs from the spirit of our previous papers on non-transitive logic, we are happy to adopt this point of view here, unblocking the way of inquiry.[4]

(c) Logical consequence

> $\Gamma \vDash^{ST\Box} \Delta$ just in case there is no interpretation M and w such that:
> for all A in Γ and all w' such that wRw': A is in $[\![w']\!]$
> and for all B in Δ, there is no w' such that B is not in $[\![w']\!]$

In other words, Δ is a consequence of Γ just in case there is no model and world w in that model where all the gammas are true at all w-accessible worlds and none of the deltas is true at any world of W (not at any w-acces sible world, just any!). Note that the conditions imposed in premises and conclusions are not each other's duals (the first pertains to accessible worlds, the second is not restricted to accessible worlds). This is the trick to break self-duality.

We require the relation R to be serial (that is, given any model, for any w there is at least a w' such that wRw'). It might look like we want to impose further restrictions on R. We might want, say, to claim that R is reflexive to grant that what it is necessary is the case. It turns out, however, that in the present logic that is already given by seriality:

Suppose that $\Box A \nvDash^{ST\Box} A$. Then there is a given w such that: for all w' such that wRw': $\Box A$ is in $[\![w']\!]$ and there is no w'' such that A is in $[\![w'']\!]$.
Now, the only way in which $\Box A$ might be true at a w without A being true *anywhere* is by w being a dead end (see Segerberg 1971, p. 93; Hughes and Cresswell 1996, p. 44).

Let us state some facts:

Fact 1 $a = b \vDash^{ST\Box} \Box(a = b)$
Fact 2 $A \vDash^{ST\Box} B \;\Rightarrow\; \vDash^{ST\Box} A \to B$
Fact 3 $\vDash^{ST\Box} (A[x/a] \wedge a = b) \to A[x/b]$
Fact 4 $a = b \wedge \diamond\neg(a = b) \nvDash^{ST\Box} \bot$

Facts 1 and 2 together show that the statement $a = b \to \Box(a = b)$ is valid. Fact 4 shows that contingent identity is consistent. Fact 3 shows that Leibniz's Law is valid in conditional form. Leibniz's Law, however, is not valid in the following form:

$$A[x/a],\ a = b \vdash A[x/b]$$

[4] "Although it is better to be methodical in our investigations and to consider the economics of research, yet there is no positive sin against logic in *trying* any theory which may come into our heads, so long as it is adopted in such a sense as to permit the investigation to go on unimpeded and undiscouraged. On the other hand, to set up a philosophy which barricades the road of further advance toward the truth is the one unpardonable offence in reasoning, as it is also the one to which metaphysicians have in all ages shown themselves the most addicted" (Peirce 1931, I. 3. §4).

since, in particular, $a = y,\ a = b \nvDash^{ST\Box} b = y$. Still, this notion of contingent identity is interesting since it preserves many of the canonical features of identity, like that of transitivity. And thus, pace Bader (2012), we conclude that contingent identity is a genuine form of identity, although we leave a proper discussion of this point for future work.

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